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Editorial

In this problem you are given two arrays of integers: $H[1,\dots,N]$ and $W[1,\dots,N]$. Your task is to decide if there is a permutation P of $[1..N]$ such that $W_1 \cdot H_{P_1}$ is **strictly greater** than $W_i \cdot H_{P_i}$ for any $2 \le i \le N$.

If you think that this problem is very easy to solve, you are right.

Let m be the index for which H_m is maximal among all other H_i. Since we want to maximize $W_1 \cdot H_{P_1}$, there is no reason to assign a value different than m to P_1.

Next, in order to satisfy the condition, we want to pair $W_2, W_3, \dots, W_N$ with remaining elements of H is such a way that $W_1 \cdot H_{P_1} > W_i \cdot H_{P_i}$ for any index $2 \le i \le N$. How to check if any such assignment exists? Well, we can act greedily here and try to construct a valid assignment.

The intuition here is that in order to minimize all values of $W_i \cdot H_{P_i}$, we can pair the greatest value from $W[2,..,N]$ with the smallest value form $H[1,\dots,N] - \{m\}$, remove them from these sets and repeat the process until are elements are paired. Now, if our assignment fulfill the condition from the statement, the answer is `"YES"`. Otherwise, we can show that there is no assignment fulfilling the condition, because exchanging any pair in our assignment cannot make the maximum value of $W_i \cdot H_{P_i}$ smaller, so the answer is `"NO"` in this case.

More formally, let's assume that our assignment does not fulfill the condition, i.e. there exists i such that $W_1 \cdot H_{P_m} \le W_i \cdot H_{P_i}$. This means that W_i cannot be paired with any $H_{P_j} \ge H_{P_i}$ either, so it has to be paired with some $H_{P_k} < H_{P_i}$. On the other hand, H_{P_i} cannot be paired with any $W_j \ge W_i$, so it has to be paired with some $W_l < W_i$. This means that values paired before with H_{P_k} and W_l cannot be paired with themselves either, so we have to assign them to other values and we will have to continue this kind of exchange, but we will never satisfy the condition.